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AN OPTIMIZED PROCEDURE FOR SEISMIC DESIGN OF TORSIONALLY UNBALANCED STRUCTURES

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SUMMARY

This paper develops an optimized procedure for the design of torsionally unbalanced structures subjected to earthquake loading, considering both the serviceability and the ultimate limit states. An optimal design eccentricity expression, in the form of design charts, and an optimal overstrength factor equation, are proposed. Results show that the recommended design procedure can result in nearly equal performance of both the rigid edge and the flexible edge elements. For a wide combination of primary system parameters, the responses of both edge elements are consistently lower than, or in the neighbourhood of, the response of the corresponding torsionally balanced reference model. The proposed procedure retains simplicity and can be easily implemented (with certain limitations) in design practice. It also has the added advantages of requiring the structure to be analysed only once for each limit state in each principal direction (as opposed to twice, in existing code torsional provisions), and results in a significantly lower overstrength factor, compared with the overstrength factors corresponding to the torsional provisions of seismic codes in the United States and Canada. The proposed procedure is also applicable to torsionally unbalanced structures with and without transverse resisting elements. © 1997 by John Wiley & Sons, Ltd.

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INTRODUCTION

Currently, torsional provisions in most seismic building codes specify two design eccentricity expressions, to be used together with the lateral forces to calculate the torques. Such a methodology has been widely employed in the seismic building codes of Europe (EC8-93[1]), Canada (NBCC-95[2]), the United States (UBC-94[3]), New Zealand (NZS:4203-92[4]) and Australia (AS 1170.4-93[5]), amongst others. The design eccentricity expressions in codes mentioned above are simple in form and easy to apply in design practice. Also, as evaluated in the companion paper,[6] torsional provisions in NBCC-95 and UBC-94 result in acceptable performance of TU structures when their torsional stiffness is moderately large or greater.

However, a number of disadvantages exist in current code torsional provisions. Firstly, the strength demand for the stiff edge element is often substantially underestimated, particularly when TU structures are torsionally flexible, as previously evaluated.[6] Secondly, the overstrength may be highly excessive. Since the strength demand of an element is decided by the more unfavourable loading resulting from applying the two design eccentricity expressions, the total strength is often much larger than the design lateral load, as shown

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in Figure 4 of the companion paper,[6] particularly when the torsional stiffness is small. Thirdly, the effects of a number of primary system parameters, such as the normalized stiffness radius of gyration, ρ_k, the force reduction factor R, and the uncoupled lateral period T_y (all defined in Reference 6), have not been considered in many codes. It has been shown[6] that these system parameters, particularly ρ_k, have critical influences on the performance of TU structures. Currently, torsional provisions in most codes mentioned above have considered the effects of one primary system parameter only, namely the normalised static eccentricity, e. Only EC8-93 (explicitly) and UBC-94 (implicitly) have taken into account the effects of another primary system parameter, ρ_k. The strength distribution in all codes is completely independent of R and T_y. Finally, due to the prescription of two design eccentricity expressions, a TU structure has to be analysed twice for each limit state and in each principal direction, and the more unfavourable strength demand has to be identified for each element. Hence, the amount of computational work required for design may be unnecessarily high, although the design eccentricity expressions themselves are simple in form.

In recent years, several analytical studies have been carried out to overcome some of the above-mentioned shortcomings in current code torsional provisions and to develop improved design procedures for TU structures. De Stefano *et al.*[7] have found that the optimized location of the Centre of Strength, (CP) is close to the mid-point between CR and CM (Figure 1). A TU system designed according to their proposals will have nearly equal inelastic responses of both the rigid edge and the flexible edge elements. However, in their study, they only considered a single global lateral force reduction factor (R) of 4 and two values of the uncoupled torsional to lateral frequency ratio (equal to ρ_k/ρ_m, where ρ_m is the mass radius of gyration of the floor slab), namely 1·0 and 1·143, both values representing TU systems having a moderate torsional stiffness. For torsionally flexible and torsionally stiff TU structures, and for those designed using a different force reduction factor, conclusions drawn in Reference 7 may not apply. This is one of the issues to be addressed in this paper.

Goel and Chopra[8] proposed a dual design approach for seismic design of TU structures. They considered design for both the serviceability limit state (SLS) and the ultimate limit state (ULS). Furthermore, for the ULS, three ductility levels, namely 2, 4 and 8, were considered. The ductility demands of TU systems were presented against a range of uncoupled lateral periods, T_y. Hence, the effect of T_y on the elastic and inelastic response of TU systems was investigated. Reference 8 proposed two design eccentricity expressions. The coefficients in their expressions are dependent on the limit state and in the case of the ULS, are also dependent on the target ductility level. However, Reference 8 has considered only one value for the uncoupled torsional to lateral frequency ratio, this system parameter being fixed as unity. Since the proposals in Reference 8 are therefore based on an analytical model which has moderate torsional stiffness, they may not be applicable to TU systems which are torsionally flexible or torsionally stiff. Furthermore, since two

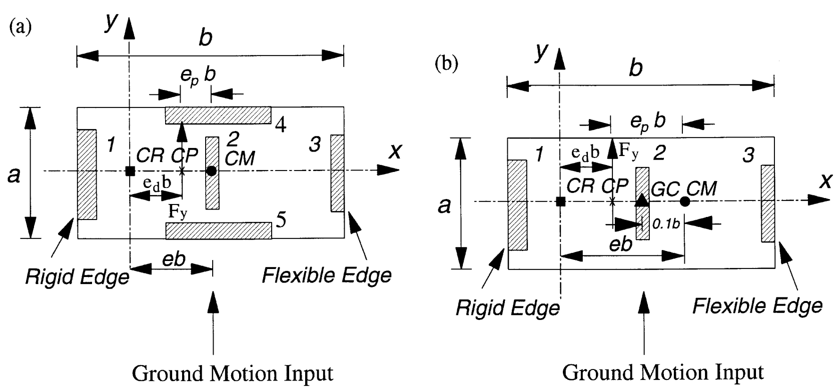

Figure 1. Plan view of (a) the stiffness-eccentric TU structural model having transverse resisting elements and (b) the mass-eccentric TU structural model

design eccentricity expressions have been proposed, some of the shortcomings discussed previously still exist in Goel and Chopra's proposals. A TU system still has to be analysed twice for each limit state in each principal loading direction. Furthermore, the resulting overstrength is still highly excessive, being higher than that resulting from UBC-94, since Reference 8 again proposed that the beneficial effect due to torsion (for the rigid edge element) should not be considered.

Mittal and Jain[9] have attempted to find the optimized design eccentricity expression, and hence the optimized location of the centre of strength, as a function of the system's normalized stiffness radius of gyration, ρ_k. Simplified equations have been proposed for design purposes. However, Reference 9 considered only one force reduction factor, $R = 5{\cdot}0$, and only one value for the uncoupled lateral period, $T_y = 1{\cdot}0$ sec. The dependency of the optimized design eccentricity expression on T_y and on R should be investigated, which are issues to be addressed in this paper. Furthermore, the structural model employed in Reference 9 has three lateral load resisting elements only, oriented parallel to the direction of ground motion input. Transverse elements are excluded. Hence, the applicability of the optimized design eccentricity expression derived on the basis of a structural model having lateral load resisting elements only, to TU structures having transverse resisting elements [Figure 1(a)] should be investigated. This is also an issue studied in the present paper.

The objective of this paper is therefore to overcome the previously discussed shortcomings in current code torsional provisions and to develop an optimised procedure for design of TU structures under seismic loading for both the SLS and the ULS. The optimization aims to achieve equal or nearly equal responses (in terms of ductility demand) of both the rigid edge and the flexible edge elements. If necessary, lateral overstrength is introduced such that the responses of both edge elements are lower than or in the neighbourhood of the response of the torsionally balanced (TB) reference system. In addition to resulting in optimized responses of TU structures, the proposed procedure is an optimized design procedure also in the sense that it reduces the overstrength factor compared with that resulting from a range of current code torsional provisions. Thus, increased efficiency and economy are achieved. The proposed procedure maintains simplicity for ease of implementation in design practice. It can be easily implemented for design of single-storey asymmetric buildings and a special class of multistorey asymmetric buildings in which the mass centres of floors lie along a vertical axis and in which the stiffness properties, namely the lateral stiffness matrices, of lateral load resisting elements parallel to the direction of the ground motion are proportional to one another. A single optimized design eccentricity expression is proposed in the form of design charts, which are easy to use in design practice and require a TU structure to be analysed only once for each limit state, in each principal loading direction. Furthermore, the proposed procedure takes into account explicitly the effects of primary system parameters, such as ρ_k and R. Also, the effect of the lateral period (related to building height) is considered implicitly. Finally, the applicability of the proposed procedure is generalized to systems having transverse resisting elements. Hence, the proposed procedure is shown to be widely applicable. Its applicability to mass-eccentric TU systems [Figure 1(b)] is also briefly studied in this paper.

STRUCTURAL MODELLING CONSIDERATIONS

The development of the proposed design procedure is based on a single-storey, stiffness-eccentric structural model, having three lateral load resisting elements only (as in Figure 1 of Reference 6). This structural model has been described in detail in the companion paper.[6] A plan view of the corresponding TU structural model having transverse elements is shown herein as Figure 1(a). The latter model is similar to that employed by Goel and Chopra in Reference 8 and is used in the latter part of the present paper to check the applicability of the proposed optimized design procedure to systems with resisting elements in both principal directions. The two transverse elements, elements 4 and 5, are assumed to be identical and fixed at the two edges of the floor slab, oriented perpendicular to the direction of the ground motion input. Elements 4 and 5 do not contribute to the system's lateral stiffness in the y-direction, K_y. However, they contribute to the system's total torsional stiffness, K_θ. It is assumed in this paper that the system's total lateral stiffnesses in the x- and y-directions,

Table I. Values of $(\rho_k)_{min}$ and $(\rho_k)_{max}$ in TU systems with transverse elements

	$b/a = 2{\cdot}5$		$b/a = 1{\cdot}5$	
e	$(\rho_k)_{min}$	$(\rho_k)_{max}$	$(\rho_k)_{min}$	$(\rho_k)_{max}$
0·1	0·30	0·53	0·40	0·59
0·2	0·32	0·50	0·42	0·57
0·3	0·32	0·44	0·42	0·52

K_x and K_y respectively, are equal. The amount of torsional stiffness provided by the two transverse elements depends on the dimension of the floor slab parallel to the y-direction, a. Therefore, the aspect ratio of the floor slab, b/a, becomes an important system parameter which influences the system's total torsional stiffness. In order to investigate the effect of this system parameter, two values, $b/a = 2{\cdot}5$ and 1·5, respectively, have been considered in the latter part of this paper. A smaller value for the aspect ratio implies that the floor slab shape is closer to square. This in turn implies that the contribution to the system's total torsional stiffness, K_θ, from the transverse elements becomes larger. The upper and the lower bounds of ρ_k of the TU model having transverse elements are given in Table I.

Since K_y and K_x are assumed to be equal, the uncoupled translational periods T_y and T_x in both principal directions are also equal. As a result, the design base shear forces F_y and F_x in both principal directions are the same. The design base shear force in the x-direction, F_x, is distributed equally between the two transverse elements. Hence, elements 4 and 5 have identical strength as well as identical stiffness. The strength distribution amongst the three lateral elements, 1, 2 and 3, respectively, is determined by carrying out a structural analysis, in which the design base shear force in the y-direction, F_y, is applied through a point on the floor slab at a distance from the CR equal to the design eccentricity, $e_d b$ [see Figure 1(a)].

Figure 1(b) illustrates the plan view of a single-storey, mass-eccentric TU structural model. It is the same in every respect as the single-storey, stiffness-eccentric TU model described in Reference 6, except that the distribution of mass is unbalanced. The Centre of Mass (CM), is located at a distance of 0·1b from the Geometric Centre (GC). CM and CR are assumed to be located at opposite sides measured from GC. However, for comparative purposes the mass inertia properties of the floor slab, namely the total mass and the mass moment of inertia about CM are assumed to be unchanged.

For all three models considered in this paper, the ground motion input and the response parameters are the same as those described in Reference 6.

OPTIMIZED DESIGN ECCENTRICITY

The term design eccentricity is defined as the distance between CR and the point on the floor slab through which the design base shear force F_y is applied for the purpose of determining the strength distribution among the lateral resisting elements. When a single design eccentricity is specified, the point described above also defines the location of the centre of strength, CP. In this case, the following relationship exists between the normalised (with respect to b) design eccentricity, e_d, and the normalised (also with respect to b) strength eccentricity, e_p

$$e_p = e - e_d \tag{1}$$

In order to develop the concept of optimized design eccentricity, it is enlightening first to investigate the influence of the normalized design eccentricity, and hence the influence of the normalized strength eccentricity, on the response of the three-element stiffness-eccentric TU structural model (see Figure 1 of Reference 6), corresponding to both limit states. Figure 2 shows the average displacement Ductility Demand (DD) of both

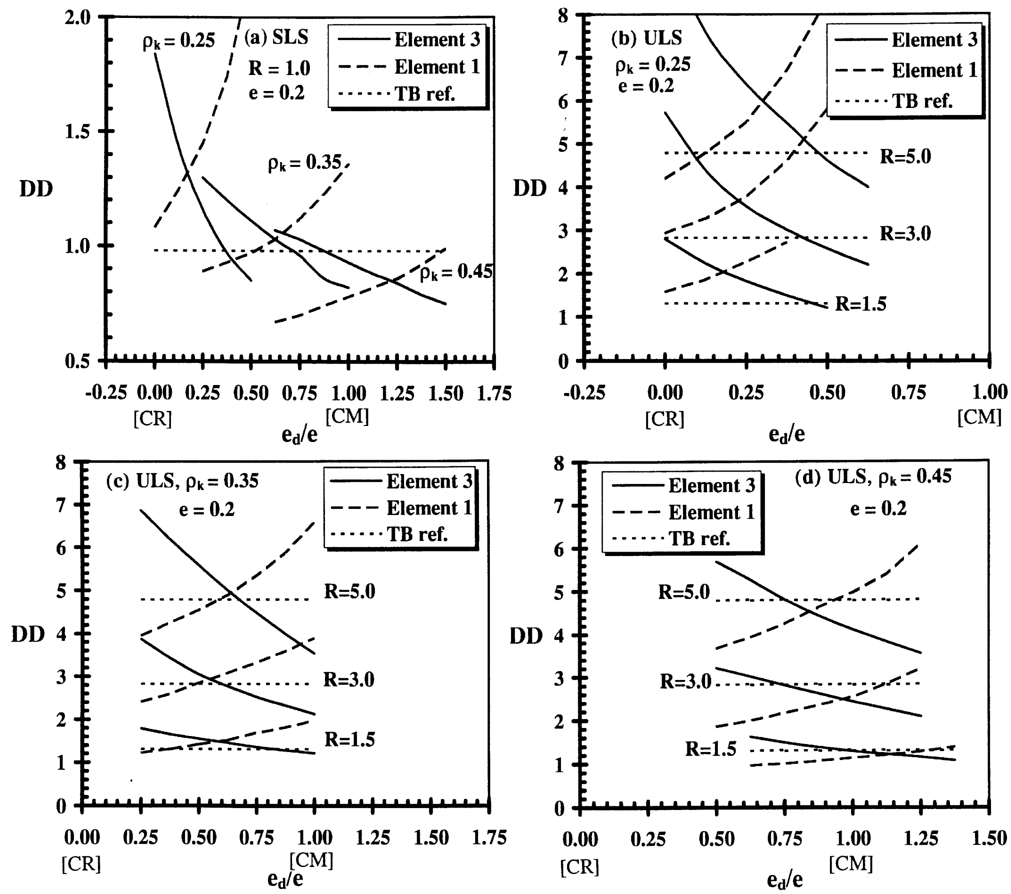

Figure 2. Variation of DD's of both edge elements with e_d/e; $e = 0{\cdot}2$ and $T_y = 1{\cdot}0$ sec

the rigid edge element (element 1) and the flexible edge element (element 3) for TU systems with $e = 0{\cdot}2$ and $T_y = 1{\cdot}0$ sec, as functions of e_d/e. The response of element 2 is insensitive to changes in e_d/e, remaining almost constant across the full range of e_d/e considered. The DD's of element 2 are slightly lower than those of the TB reference system and hence for brevity are not shown in this paper. When e_d/e increases, the DD's of element 1 increase monotonically and those of element 3 decrease monotonically. The intersection point of curves corresponding to DD's of elements 1 and 3 defines the optimized location of CP, and hence defines the optimized value of the normalized strength eccentricity, e_{po}, and from equation (1) the optimized value of the normalised design eccentricity, e_{do}. When a TU structure is designed such that $e_d = e_{do}$, the response of the TU system will be optimal, with the DD's of both edge elements being equal. Results corresponding to other e and T_y values show similar trends and hence are not shown herein, although the co-ordinates of the intersection points differ from those given in Figure 2.

The optimized normalized design eccentricity, e_{do}, depends on ρ_k, e, R and T_y, with ρ_k and e found from extensive studies to be the most influential parameters. Generally, the value of e_{do}/e increases with the increase in ρ_k (Figure 2). In the neighbourhood of the intersection point, the curves are steep when ρ_k is small or R is large. However, the curves tend to flatten when ρ_k is large or R is small. This observation implies that in the former case, the responses of both edge elements are sensitive to the deviation of the design eccentricity

away from e_{do} and hence the optimized design eccentricity is closely defined. In the latter case, the responses of both edge elements are less sensitive to the deviation of the design eccentricity from e_{do}, remaining little changed when the design eccentricity is in the neighbourhood of e_{do}.

Compared with the DD's of the TB reference model, the DD's corresponding to the intersection points are higher when ρ_k is small (0·25, the lower bound of ρ_k of the three-element TU structural model, as discussed in Reference 6), approximately equal when ρ_k is moderately large (0·35), and slightly lower when ρ_k is large (0·45). This observation suggests that in order for the responses of both edge elements to be lower than, or in the neighbourhood of, that of the TB reference model, a certain amount of overstrength is required for torsionally flexible TU systems, whilst for torsionally moderately stiff and very stiff TU systems, overstrength is not needed, provided that e_{do} (or a value very close to it) is employed in determining the element strength distribution. It can be further concluded that if adequate overstrength is provided, any design eccentricity values in the neighbourhood of e_{do} will result in responses of both edge elements being lower than or in the neighbourhood of that of the TB reference model. The higher the overstrength, the wider the range in which e_d could be permitted safely to deviate from e_{do}.

For moderately torsionally stiff TU systems ($\rho_k = 0{\cdot}35$), e_{do}/e is indeed close to 0·5, agreeing with results obtained in Reference 7. However, for torsionally flexible ($\rho_k = 0{\cdot}25$) and torsionally stiff ($\rho_k = 0{\cdot}45$) TU systems, the value of e_{do}/e varies over a wide range, from close to 0.0 in the former case to close to 1·5 in the latter case. It is evident that the influence of ρ_k on e_{do}/e is critical and hence should be considered in developing an optimized procedure for seismic design of TU structures.

In order to identify the values of the optimized normalized design eccentricity, e_{do}, a one-dimensional search has been carried out. For a given set of system parameters, e, ρ_k, R, and T_y, which together completely define a three-element TU system, the normalized design eccentricity, e_d, has been varied between $-0{\cdot}2e$ and $+1{\cdot}8e$ at intervals of $e/20$. The normalized static eccentricity e is given three values, 0·1, 0·2 and 0·3, respectively. The normalized stiffness radius of gyration ρ_k ranges from 0·25 to 0·45 when e equals 0·1 and 0·2, and ranges from 0·25 to 0·4 when e equals 0·3. The force reduction factor R is unity in the case of the SLS and is taken equal to 1·5, 3·0 and 5·0 in the case of the ULS. The uncoupled lateral period T_y is given three values, namely 0·5, 1·0 and 2·5 sec. For all combinations of the above system parameters, the DD's of both edge elements have been computed and the intersection points have been identified. The resulting values of e_{do}/e are shown in Figure 3 as functions of ρ_k and R, for $T_y = 0{\cdot}50$ sec only. Results corresponding to $T_y = 1{\cdot}0$ and 2·5 sec show the same trends although values may differ. Hence, they are not presented herein for brevity. The period dependency of e_{do} is the subject of discussion in the following section.

It may be observed from Figure 3 that the value of e_{do}/e generally increases with the increase of ρ_k and decreases with the increase of R. This observation may be predicted by the results obtained in the companion paper.[6] It has been shown in Reference 6 that the DD's of element 1 decrease with the increase of ρ_k, while the DD's of element 3 increase with the increase of ρ_k. Hence, in order to achieve equal DD's of elements 1 and 3, e_{do}/e must be increased with the increase of ρ_k, such that the strength of element 1 is reduced and the strength of element 3 is increased. It has also been concluded in Reference 6 that the displacement ductility ratio, Γ_{DD}, of element 1 increases significantly with the increase of the force reduction factor, R, whilst Γ_{DD} of element 3 is little influenced by R. Consequently, when R increases, in order to achieve equal DD's and hence Γ_{DD}'s for both elements 1 and 3, e_{do}/e must be reduced, such that the centre of strength CP is moved towards the centre of rigidity CR, increasing the proportion of strength allocated to element 1 and correspondingly reducing the proportion of strength allocated to element 3.

PERIOD DEPENDENCY OF THE OPTIMIZED DESIGN ECCENTRICITY

The optimized design eccentricity is period dependent, although its trend of variation with ρ_k and R is similar to that in Figure 3, for all three period values considered in this study. Figure 4 illustrates the period dependency of e_{do}/e for TU systems having a moderate ductility capacity, $R = 3{\cdot}0$. Results corresponding to $R = 1{\cdot}5$ and $R = 5{\cdot}0$ show similar trends and are hence not presented herein. It may be concluded that the

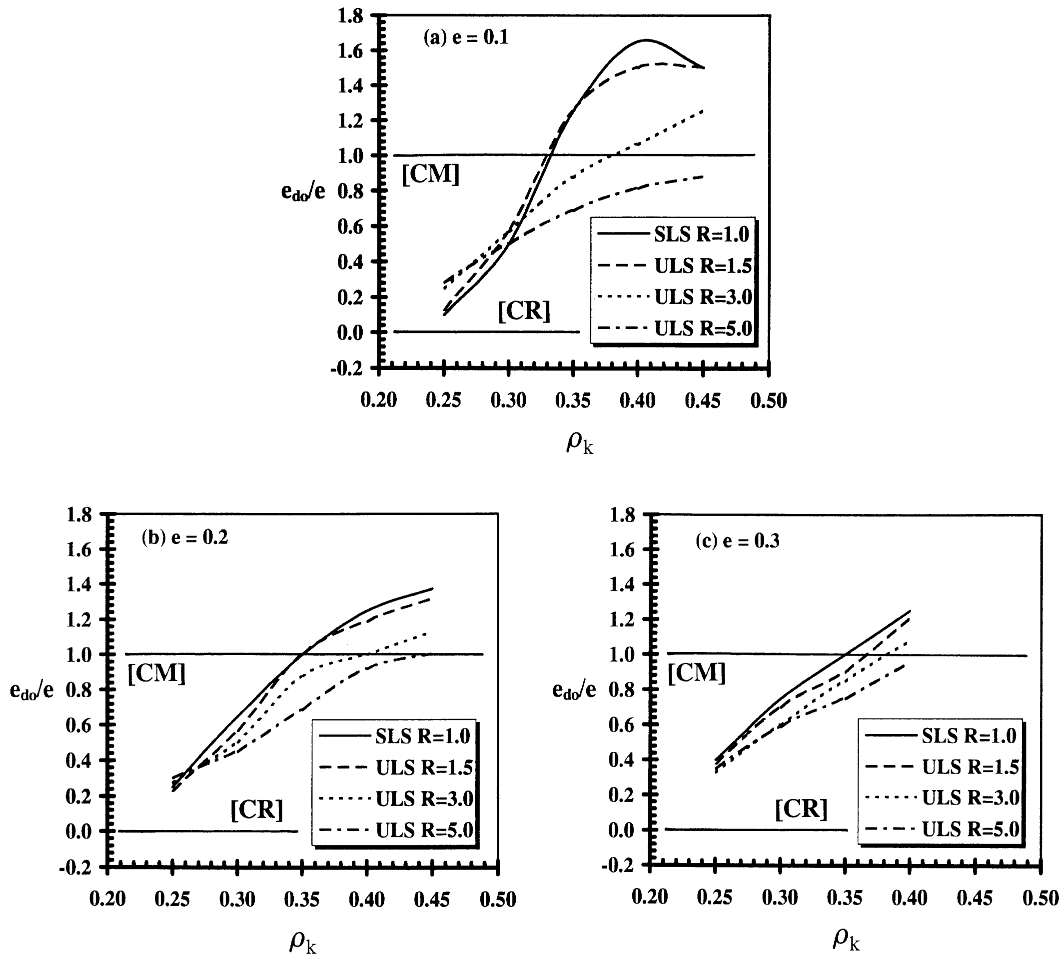

Figure 3. Values of e_{do}/e; $T_y = 0{\cdot}50$ sec

dependency of e_{do}/e on the uncoupled lateral period, T_y, is significant. The value of e_{do}/e in long period ($T_y = 2{\cdot}5$ sec) TU systems is substantially lower than that in short period ($T_y = 0{\cdot}5$ sec) TU systems. The value of e_{do}/e corresponding to intermediate period ($T_y = 1{\cdot}0$ sec) TU systems is generally between those corresponding to short- and long-period TU systems, being close to that of short-period TU systems when ρ_k is small and being close to that of long-period TU systems when ρ_k is large.

The above observation is also consistent with results obtained in the companion paper.[6] It has been shown in Reference 6 that the displacement ductility ratio Γ_{DD} of element 1 is significantly larger in long period TU systems than in short period TU systems. In contrast, Γ_{DD} of element 3 is somewhat lower in long-period TU systems than in short-period TU systems. Consequently, in order to achieve equal DD's and hence Γ_{DD}'s of both elements 1 and 3, the value of e_{do}/e must be smaller in long-period TU systems than in short-period TU systems, such that the centre of strength CP is shifted towards the centre of rigidity CR, thus increasing the strength allocated to element 1 and reducing the strength allocated to element 3.

Results shown in Figures 3 and 4 suggest that the optimized normalized design eccentricity, e_{do}, is dependent on all four primary system parameters, namely e, ρ_k, R and T_y. Among these parameters, the dependency of e_{do} on the first two is the strongest and most critical, while the dependency of e_{do} on T_y is

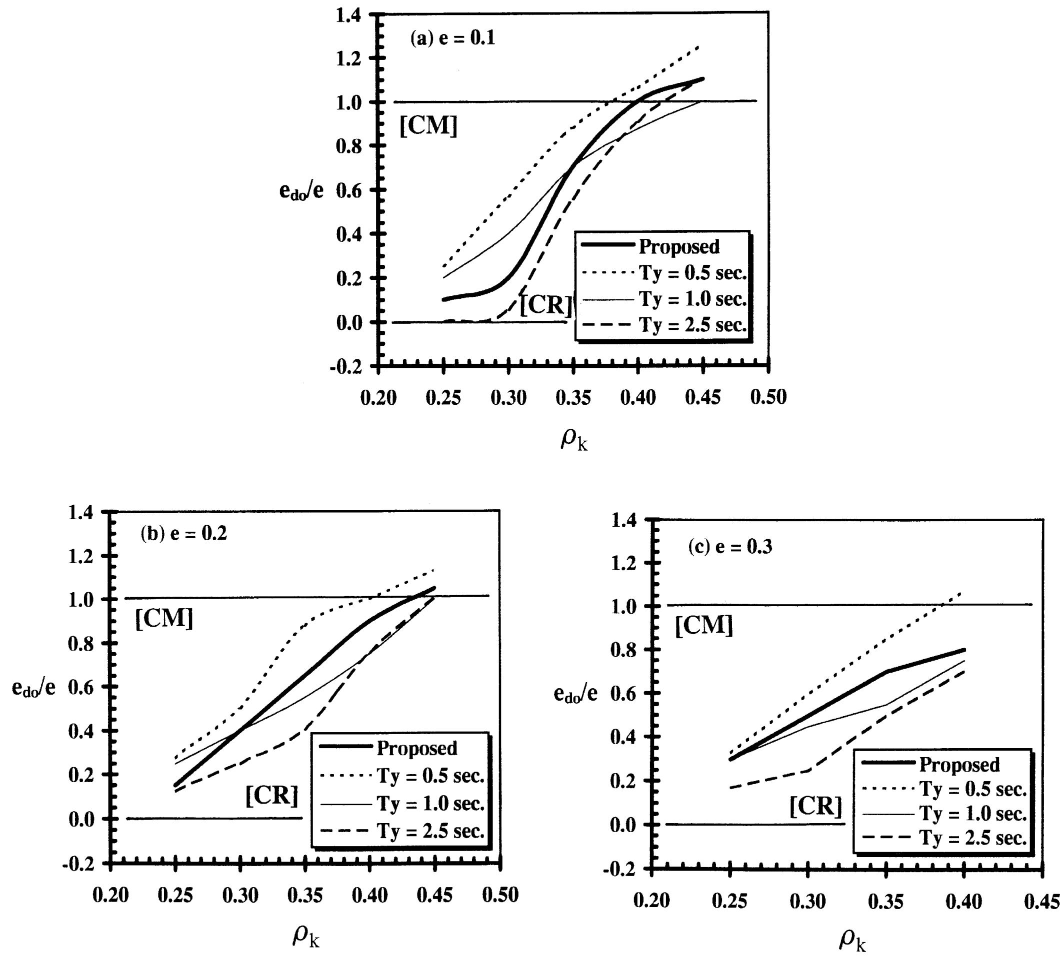

Figure 4. Period dependency of e_{do}/e; $R = 3{\cdot}0$

somewhat less strong and hence less critical. For purposes of simplicity and ease of implementation in design practice, it is advantageous to have design eccentricity expressions which are as close as possible to the optimized design eccentricity and at the same time depend on as few as possible system parameters. Therefore, it is desirable and practical to propose nearly optimized design eccentricity expressions which are period independent by adopting an 'averaged' curve for design purposes. As illustrated in Figure 4, the curve corresponding to results obtained for short period TU systems approximately sets the upper bound and that corresponding to long-period TU systems approximately sets the lower bound of the optimized design eccentricity, across the full range of the uncoupled lateral period. Hence, an 'averaged' curve for the design eccentricity, which is period independent and in the neighbourhood of the optimized design eccentricity, can be determined based on the criterion that it is more or less at equal distances to the upper and lower bounds.

Superimposed onto Figure 4 (the bold solid lines) are the proposed 'averaged' curves for design purposes when $R = 3{\cdot}0$. In the same manner, the 'averaged' curves corresponding to other R values of the ULS (namely $1{\cdot}5$ and $5{\cdot}0$), as well as the curve corresponding to the SLS ($R = 1{\cdot}0$), have also been determined (see Figure 5). As a result of the above mentioned 'averaging' procedure, the approximately optimised normalized design eccentricity, simply referred to as e_d, is now dependent on three primary system parameters, namely e, ρ_k and

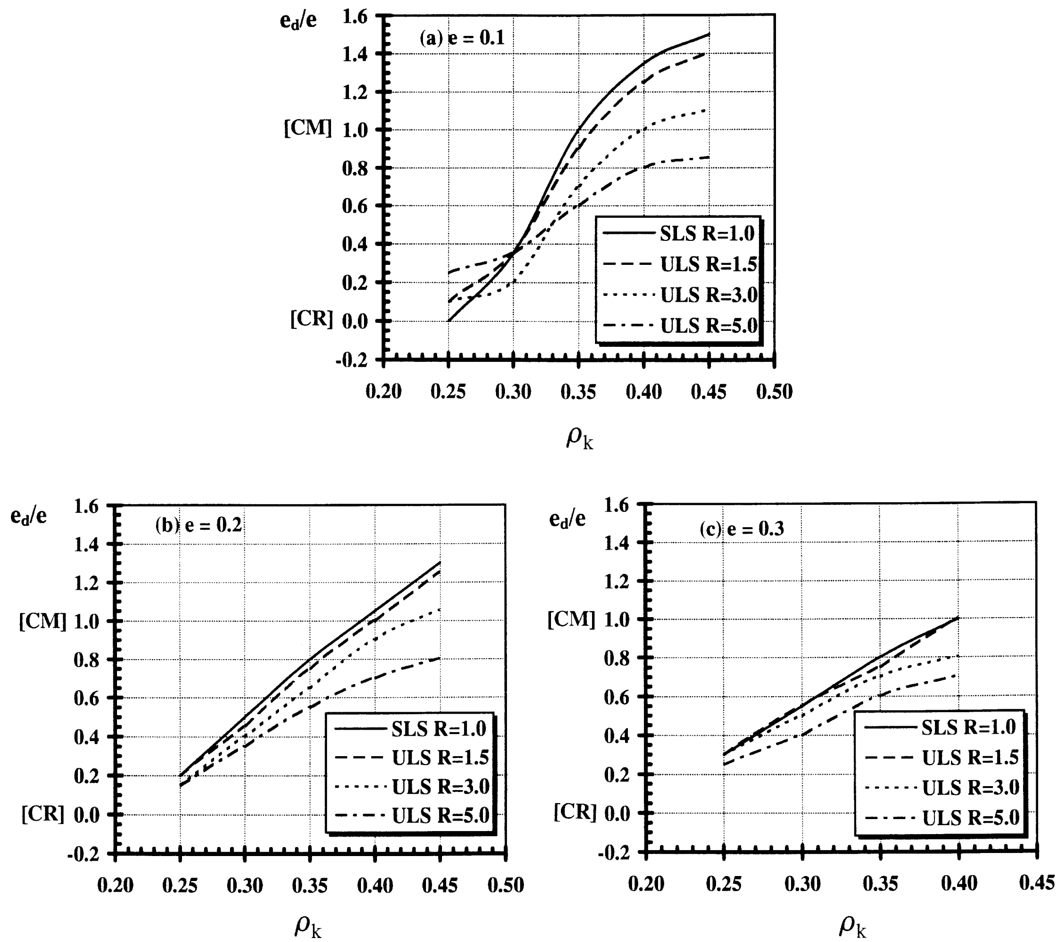

Figure 5. Design charts for the approximately optimised normalised design eccentricity, e_d

R. Hence, e_d/e, identified by the procedure described above, can be presented in the form of design charts for a range of eccentricities, as functions of ρ_k and R only, as shown in Figure 5 for $e = 0{\cdot}1$, $0{\cdot}2$ and $0{\cdot}3$. For codified design purposes, the set of design charts should be presented in smaller increments of e, such as 0.05 or even less. Therefore, once the design procedure has been fully established, further research work should be carried out to propose a complete set of charts for the optimised design eccentricity.

Due to the procedure of 'averaging', the proposed normalized design eccentricity, e_d, given by the design charts (Figure 5) will deviate from, although remaining in the neighbourhood of, the optimized value, e_{do}. As a result, the DD's of elements 1 and 3 will differ and the DD of one of the two elements may often exceed that of the corresponding TB system, as illustrated in Figure 2. However, as previously pointed out, if adequate overstrength is provided, the DD's of both edge elements can be reduced to values lower than or in the neighbourhood of the DD's of the corresponding TB system. This issue is discussed in the following section.

OVERSTRENGTH

As shown in Figure 2 and discussed previously, when compared with the DD's of the TB reference system, the DD's corresponding to the intersection points, namely when e_d is equal to e_{do}, are larger when ρ_k is small,

approximately equal in value when ρ_k is moderately large and slightly lower when ρ_k is large. It is enlightening at this stage to examine the displacement ductility ratio Γ_{DD}, which is defined as the ratio between the DD's corresponding to the intersection points of TU systems and those of the corresponding TB reference systems. Figure 6 presents Γ_{DD}'s corresponding to long period ($T_y = 2{\cdot}5$ sec) and short-period ($T_y = 0{\cdot}5$ sec) TU systems for all three eccentricity values considered, namely $e = 0{\cdot}1$, $0{\cdot}2$ and $0{\cdot}3$. Results corresponding to $T_y = 1{\cdot}0$ sec show similar trends and are hence not presented herein.

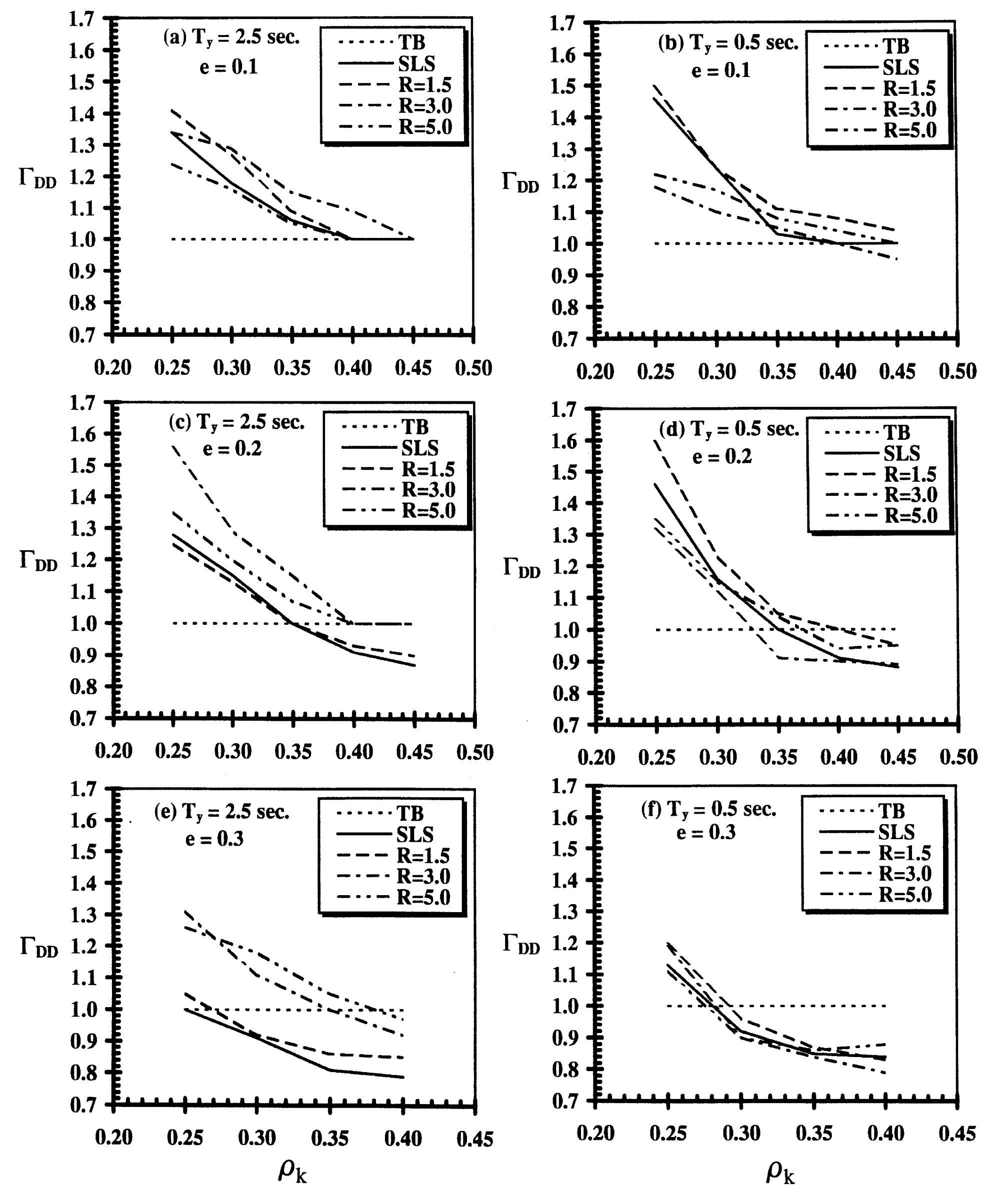

Figure 6. Values of the average displacement ductility ratio, Γ_{DD}

The displacement ductility ratio, Γ_{DD}, decreases with the increase in ρ_k and e. It is also seen to be influenced by the force reduction factor, R. The value of Γ_{DD} indicates approximately the overstrength factor which is needed in TU systems in order to have the DD's corresponding to the intersection points of TU systems to be equal to the DD's of the TB reference systems. Furthermore, when the 'averaged' period independent design eccentricity as proposed in the form of design charts in Figure 5 is employed to determine the strength distribution in TU systems, an additional amount of overstrength is needed in order to have the DD's of both edge elements to be lower than or in the neighbourhood of the DD's of the TB reference system. This latter objective is achieved in this study by increasing the design base shear force by a single overall overstrength factor. The criterion adopted for determining the overstrength factor is to result in the DD's of both edge elements to be lower than, or no more than 10 per cent higher than, the DD's of the corresponding TB reference system. A 10 per cent margin above the DD's of the corresponding TB reference system is tolerated in this study, since within this margin the DD of an element in the TU system may be regarded as in the neighbourhood of the DD of the corresponding TB reference system and hence may be considered satisfactory.

The trend of variation of Γ_{DD} as a function of ρ_k, as shown in Figure 6, suggests that the overstrength factor may vary linearly when ρ_k ranges from its lower bound to a moderately large value, then remaining constant when ρ_k is large. Furthermore, for simplicity and ease of implementation in design practice, it is advantageous to have the overstrength factor to be dependent on ρ_k only. Therefore, this paper recommends that the overstrength factor, O_s, be determined as follows:

$$O_s = \begin{cases} -3{\cdot}33\rho_k + 2{\cdot}33, & 0{\cdot}25 \leqslant \rho_k \leqslant 0{\cdot}40 \\ 1{\cdot}0, & \rho_k > 0{\cdot}40 \end{cases} \tag{2}$$

The recommended overstrength factor therefore has its largest value of 1·5 at $\rho_k = 0{\cdot}25$, decreasing linearly to unity at $\rho_k = 0{\cdot}40$, and thereafter remaining constant, as shown by the bold solid line in Figure 7. For the purpose of comparison, the overstrength factors corresponding to systems designed according to the torsional provisions of EC8-93, UBC-94 and NBCC-95 have been superimposed onto Figure 7, for systems with moderate and large eccentricity. When calculating the overstrength factors corresponding to these three

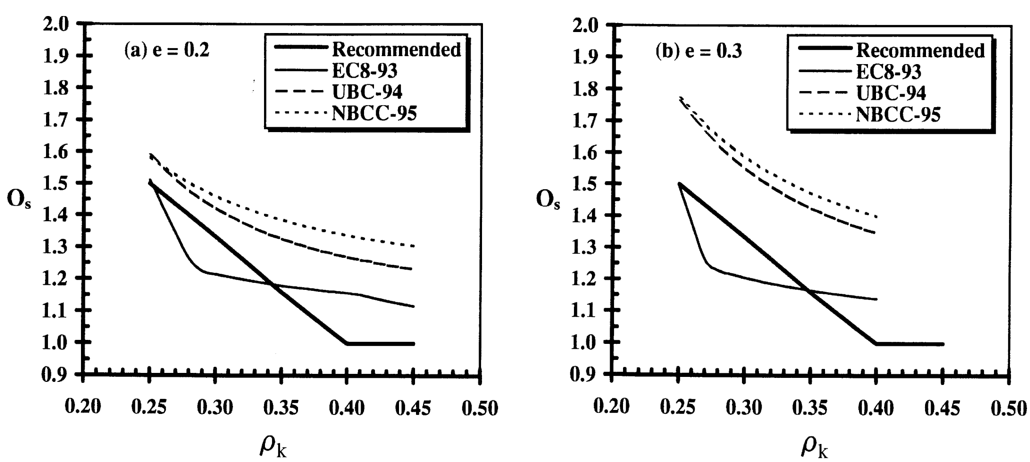

Figure 7. The recommended overstrength factor, compared with the overstrength factors resulting from the torsional provisions of EC8-93, UBC-94 and NBCC-95

codes, the code accidental torsional provisions are interpreted in the same manner as the second approach, Case 2, as outlined in the companion paper.[6] The advantage of the recommended overstrength factor is clearly demonstrated in Figure 7. The overstrength factor recommended in this paper is substantially lower than that corresponding to UBC-94 and NBCC-95, across the full range of ρ_k. Compared with the overstrength factor resulting from EC8-93, the recommended values are marginally higher when ρ_k ranges from small to moderate, and are marginally lower when ρ_k ranges from moderate to large.

OPTIMIZED PROCEDURE FOR DESIGN OF TU STRUCTURES FOR BOTH LIMIT STATES

Although a common seismic design philosophy has been accepted world-wide, its implementation varies from country to country. Generally, two approaches have been adopted by national codes. The first approach, which may be termed a one-phase design procedure, is to primarily safeguard against structural failure and the consequent loss of life. It does not explicitly attempt to limit damage to maintain functioning of buildings when they are subjected to relatively more frequent minor and moderate earthquake ground motions. Hence this procedure only specifies design spectra for the ULS design. Designers are not required to check explicitly that the SLS requirements are satisfied. This one-phase design procedure has been adopted by EC8-93[1] in Europe, UBC-94[3] in the United States and NBCC-95[2] in Canada. The second approach, which may be termed a two-phase design procedure, explicitly specifies design spectra for both limit states. Designers are required to check explicitly that the requirements for both limit states are satisfied. This two-phase design procedure has been adopted by BCJ-91[10] in Japan, NZS-92[4] in New Zealand and GBJ-89[11] in China.

Although the lateral seismic force provisions of some building codes have taken into account the differences between the two limit states, none of the above-mentioned codes has considered the influence of the two different limit states, and in the case of the ULS, the influence of the force reduction factor, on the horizontal distribution of strength. Therefore, torsional provisions in seismic building codes currently are not dependent on the design limit state concerned and are hence not dependent on the force reduction factor. Results in the companion paper[6] have shown that the force reduction factor, R, influences the response of the stiff edge element significantly. It has also been shown in Reference 6 that when R increases, code torsional provisions become increasingly overconservative in estimating the strength demand of the flexible edge element, while they become increasingly unconservative in estimating the strength demand of the rigid edge element. In order to overcome these shortcomings in current code torsional provisions, this paper recommends an optimized, two-phase procedure for design of TU structures for both limit states. In this procedure, not only the design seismic lateral base shear force, but also the design eccentricity expressions, are dependent on the limit state concerned, and in the case of the ULS, the design eccentricity expressions are dependent on the design value of the force reduction factor, R. For each element, the more unfavourable strength demand resulting from the SLS design and the ULS design should be identified and used for sizing and detailing the element considered. The recommended procedure may be summarized by the sequence of steps given below.

Step 1: The TU structure's fundamental uncoupled lateral period, T_y, is estimated (ignoring torsion) using the simplified methods suggested in building codes.

Step 2: The seismic lateral base shear forces for both limit states are calculated, according to the design spectra specified in building codes for the SLS (for instance the design spectrum for the SLS in the New Zealand building code NZS:4203-92[4]) and for the ULS. In the case of the SLS, the base shear force is dependent on the period T_y estimated in Step 1. In the case of the ULS, the base shear force is additionally dependent on the force reduction factor, R. The base shear forces thus calculated are already a conservative estimate, since torsion has been ignored at this stage.

Step 3: The two system parameters, e and ρ_k, are calculated. For single-storey asymmetric buildings, the procedure for calculating e and ρ_k is straightforward and well established. For the special class of multistorey asymmetric buildings in which the mass centres of floors lie along a vertical line and in which the stiffness properties of lateral load-resisting elements parallel to the y-axis are proportional to one another, a simple procedure for calculating e and the uncoupled torsional to lateral frequency ratio, Ω, has been developed by Hejal and Chopra.[12] The value of ρ_k can then be calculated by using the relationship $\Omega = \rho_k/\rho_m$.

Step 4: The lateral overstrength factor, O_s, is calculated using equation (2), or alternatively is read from Figure 7. O_s is dependent only on the system's normalized stiffness radius of gyration, ρ_k.

Step 5: The modified base shear forces for both limit states are calculated by multiplying the base shear forces obtained in Step 2 by the overstrength factor, O_s, obtained in Step 4.

Step 6: The design eccentricity values for both limit states are determined by reading e_d/e from the design eccentricity charts shown in Figure 5 (plus others to be developed for intermediate eccentricity values as appropriate). The values of e_d/e depend on ρ_k, together with the limit state concerned. In the case of the ULS, e_d/e also depends on the design force reduction factor, R.

Step 7: For each limit state, the strength demand of each resisting element is obtained by applying the corresponding modified base shear force, obtained in Step 5, through a point on the floor deck at a distance from the centre of rigidity, CR, equal to the corresponding design eccentricity, obtained in Step 6, and then carrying out an elastic, static structural analysis.

The above listed steps have been used to determine the strengths of resisting elements of the TU structural model having three lateral resisting elements only (Figure 1 of Reference 6) and having various combinations of primary system parameters. A series of dynamic response analyses of such designed TU structural models subjected to the chosen ensemble of earthquake records[6] have been carried out. The resulting DD's of elements 1 and 3 are shown in Figures 8 and 9 as functions of ρ_k, for all combinations of three periods, three eccentricity values, and both limit states (in the case of the ULS, three force reduction factor values), considered in this paper. Figure 8 (SLS) and Figure 9 (ULS) clearly illustrate the closely optimized response of both edge elements resulting from the proposed design procedure. Not only are the DD's of both edge elements in TU systems now lower than, or in the neighbourhood of, those of the TB reference system, but the DD's of both edge elements are also nearly equal in value and more or less constant across the full range of ρ_k. Hence, for both limit states, the proposed optimised design procedure leads to consistent and acceptable responses of both edge elements in TU systems having a wide range of primary system parameters.

In actual design practice, accidental torsional effects, which have not yet been considered explicitly, should be taken into account at this stage by increasing the strength of each element for each limit state appropriately. Finally, for each element, the larger strength demand resulting from the two limit state designs should be used for sizing and detailing the element under consideration. Therefore, the following additional two steps of analysis should be carried out in actual design practice.

Step 8: The accidental torsional effects are considered by increasing the strength demand of each element appropriately, for each limit state.

Step 9: For each resisting element, the more unfavourable strength demand resulting from the SLS design and the ULS design is identified and used for sizing and detailing the element concerned.

It should be noted that the explicit consideration of accidental torsional effects is beyond the scope of this paper. In a recent study, De La Llera and Chopra[13] have proposed a methodology to consider the accidental torsional effects in elastic (corresponding to the SLS) design of TU structures. Their approach has a number of advantages. Firstly, the increase in element strengths required by their method has a well-established

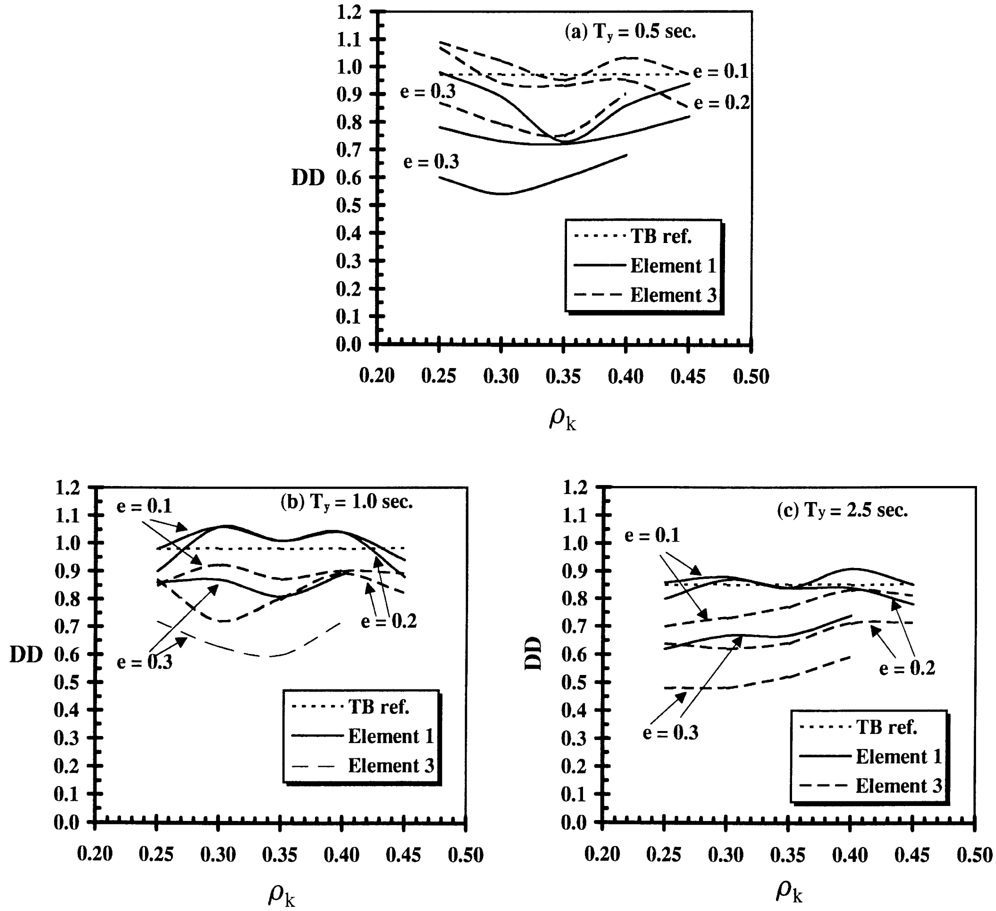

Figure 8. Performance of the 3-element TU structural model in the SLS

probability of exceedance. Secondly, there is no need to carry out structural analyses in considering the accidental torsional effects. Hence, this paper recommends that the methodology proposed in Reference 13 be employed in Step 8 to consider the accidental torsional effects in the SLS. The accidental torsional effects in the inelastic (corresponding to the ULS) response of TU structures remains an issue to be resolved by future studies.

GENERALIZATION OF THE PROPOSED DESIGN PROCEDURE

The closely optimized procedure for design of TU structures, as proposed in the preceding section is based on a three-element stiffness-eccentric TU structural model having lateral load resisting elements only. Transverse resisting elements have been excluded. The horizontal distribution of mass has been assumed to be uniform. Questions arise, concerning the applicability of the above-proposed design procedure to realistic structural systems having transverse resisting elements or those systems having asymmetric distribution of mass. This section addresses these two issues in order to generalize the applicability of the optimized design procedure.

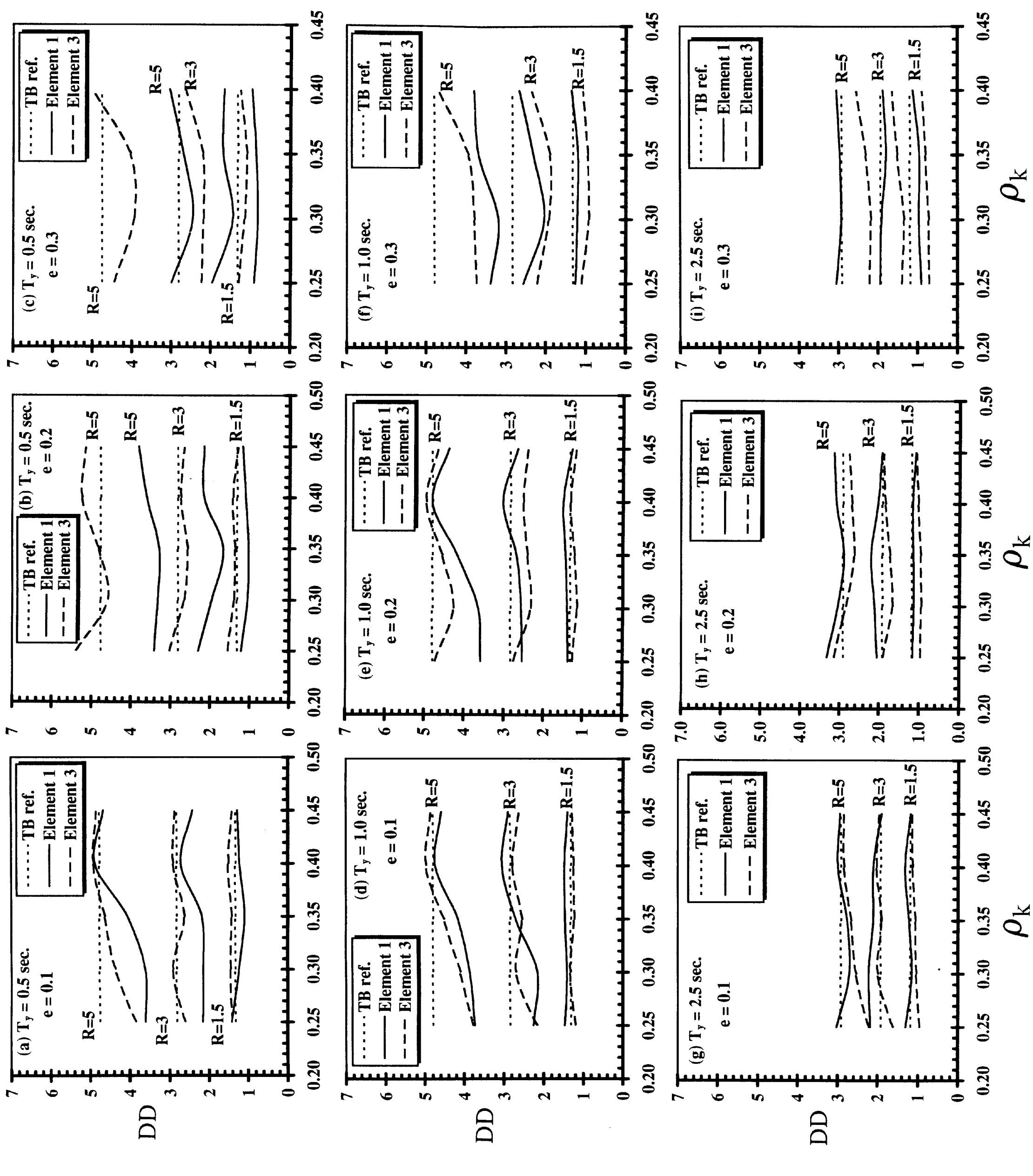

Figure 9. Performance of the 3-element TU structural model in the ULS

1. TU systems having transverse resisting elements

Most buildings in engineering practice have resisting elements oriented in two orthogonal directions and are subjected to two orthogonal horizontal components of ground motion. Although the transverse elements do not contribute to resistance in the lateral direction, they provide torsional resistance if they remain elastic when loaded by the torsional response of the structure and the ground motion component parallel to the transverse direction. The capacity of torsional resistance which can be provided by the transverse elements depends on the intensity of the ground motion component parallel to the transverse direction, relative to the total strength in the transverse direction. When the transverse elements are loaded into the inelastic range during most of the time of the response, the torsional resistance contributed by the transverse elements will be very small. As a result, the system's lateral and torsional responses can be accurately modelled by considering the lateral elements and the lateral component of the ground motion only, as if the transverse elements and the transverse component of the ground motion did not exist. Recent full bidirectional studies (References 14 and 15) have confirmed the validity of this approach.

On the other hand, if the intensity of the transverse component of the ground motion is weak relative to the total strength of the transverse elements, these elements will remain elastic during most of the duration of the response. Consequently, the torsional resistance contributed by these elements is significant and cannot be neglected. In this case, the system's lateral and torsional responses can be accurately modelled by including both lateral and transverse elements in the structural model and considering the lateral component of the ground motion only, as if the transverse component of the ground motion did not exist.

The above two structural modelling considerations provide the two extremes of simplification. The building's actual response will lie somewhere between these two extremes, as suggested by De La Llera and Chopra.[15] The former case has been dealt with in the preceding sections. The latter is examined in this section.

The TU structural model having transverse elements [Figure 1(a)] has been described previously. The optimized design procedure proposed in the preceding section (Step 1–Step 7) is again used to determine the strengths of the lateral elements, namely elements 1–3. The torsional stiffness provided by the two transverse elements, namely elements 4 and 5, has been taken into account when carrying out the static elastic structural analysis outlined in Step 7. The strengths of the two transverse elements are determined in the manner described previously.

Since the TU structural model having transverse elements is torsionally stiffer compared with the three-element TU structural model, the upper bound of ρ_k of the former model is larger than that of the latter model (see Table 1 and Figure 5). Hence, in Step 6, when reading the values of the design eccentricity from the design charts shown in Figure 5, the curves must be extrapolated beyond the corresponding upper bounds of ρ_k of the three-element TU model. It is assumed in this paper that the value of e_d/e remains constant, being equal to its value at the upper bound of ρ_k, when ρ_k is larger than the upper bound given in Figure 5. This assumption is supported by results shown in Figure 2, which indicate that when ρ_k is large (0·45), the responses of both edge elements are less sensitive to the deviation of e_d from e_{do}. Furthermore, results in Figure 2 suggest that when ρ_k is large (0·45), e_d could be permitted to deviate from e_{do} within a wide range, in which the resulting DD's of both edge elements are lower than the DD of the corresponding TB reference system. Consequently, the above-mentioned extrapolation is valid and is expected to lead to satisfactory performance of torsionally very stiff TU systems having values of ρ_k larger than the upper bound of ρ_k in the three-element TU model.

A series of dynamic response analyses of the TU system having transverse elements, subjected to the ensemble of selected earthquake records, have been carried out for all combinations of primary system parameters, including two values of the aspect ratio, b/a, of the floor slab (namely 2·5 and 1·5). Figures 10 ($b/a = 2{\cdot}5$) and 11 ($b/a = 1{\cdot}5$) present the DD's of both edge elements of this type of TU system, as functions of ρ_k, for intermediate period ($T_y = 1{\cdot}0$ sec) TU systems. Results corresponding to short-period ($T_y = 0{\cdot}5$ sec) and long-period ($T_y = 2{\cdot}5$ sec) TU systems show the same trends and hence for brevity are not presented

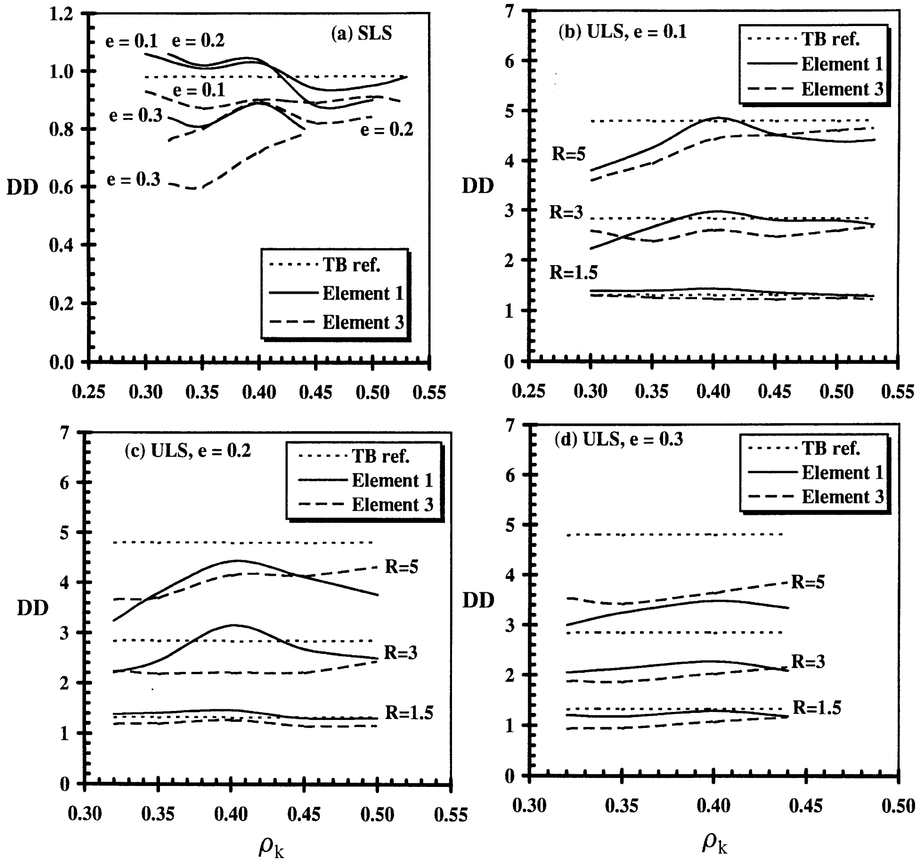

Figure 10. Performance of the stiffness-eccentric TU structural model having transverse resisting elements, in both limit states; $b/a = 2{\cdot}5$

herein. For both values of the aspect ratio, the proposed design procedure leads to closely optimized responses of both elements 1 and 3. The DD's of both elements 1 and 3 are more or less constant and are nearly equal in value across the full range of ρ_k realistically expected in actual buildings. The DD's of both elements 1 and 3 are lower than, or in the neighbourhood of, the DD's of the TB reference system. Consequently, the proposed optimized design procedure has been shown to be also applicable to TU systems in which the torsional resistance contributed by transverse elements must be considered in the structural modelling.

2. Mass-eccentric TU systems

Some previous studies (References 16 and 17) have reported that Mass-Eccentric Systems (MES) and Stiffness-Eccentric Systems (SES) respond differently in the inelastic range. The displacement ductility demand of the stiff edge element in a MES is significantly larger than that in a SES, while the displacement ductility demand of the flexible edge element in a MES is significantly smaller than that in a SES. Hence, design guidelines derived based on a SES may lead to poor response of a MES, and vice versa.

The chosen three-element mass-eccentric TU structural model has been described previously. In order to examine the applicability of the proposed design procedure to this type of TU structure, the optimized design

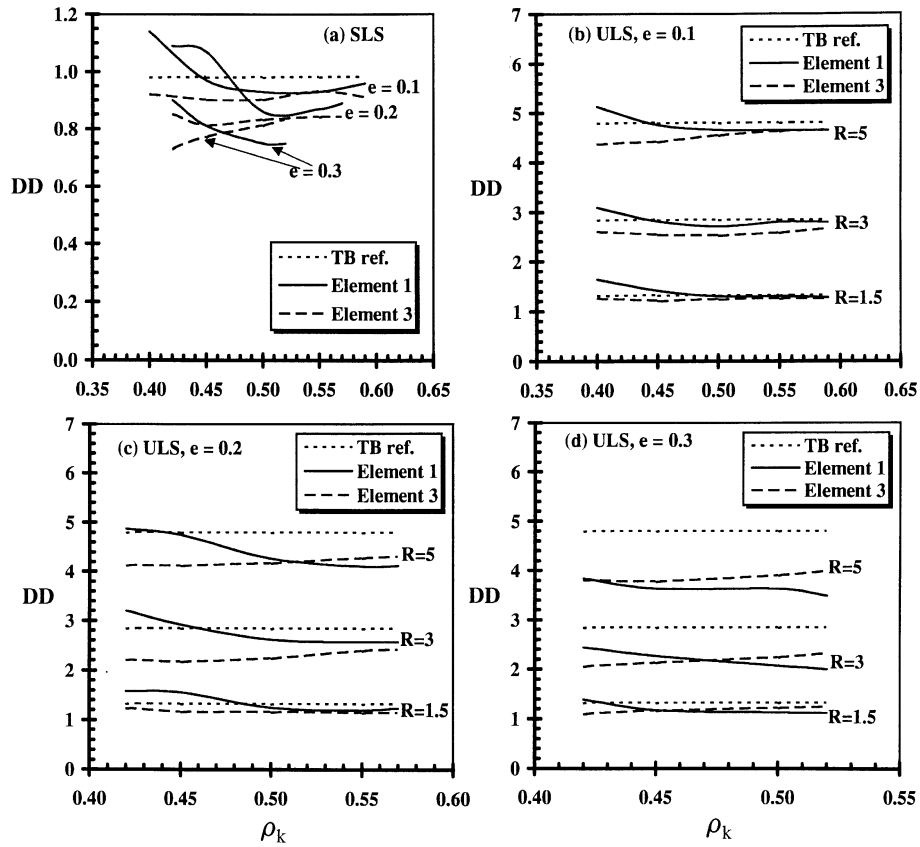

Figure 11. Performance of the stiffness-eccentric TU structural model having transverse resisting elements, in both limit states; $b/a = 1{\cdot}5$

procedure proposed in the preceding section (Step 1 to Step 7) is once again used to determine the strengths of the lateral elements. A series of dynamic response analyses of mass-eccentric TU systems, subjected to the ensemble of selected earthquake records, have been carried out for all combinations of primary system parameters. Figure 12 presents the DD's of both elements 1 and 3 of this type of TU system as functions of ρ_k, for an intermediate period ($T_y = 1{\cdot}0$ sec) TU system. Results shown in Figure 12 confirm conclusions drawn in References 16 and 17. The DD's of the stiff edge element, element 1, are often much larger than the DD's of the TB reference system. In particular, when e is large ($e = 0{\cdot}3$) and R is small to moderate ($R = 1{\cdot}0$ [SLS], 1·5 and 3·0 [ULS]), the former may be up to twice the value of the latter. Clearly, a design procedure derived based on a SES is not fully applicable to a MES, as concluded by Tso and Ying.[17] Consequently, further research is needed to achieve a design procedure applicable to a MES.

CONCLUDING REMARKS

This paper has developed a closely optimized procedure for design of TU structures subjected to earthquake loading for both the serviceability and the ultimate limit states. The procedure aims to result in nearly equal responses of both edge elements. Design charts for an optimized, 'averaged' (over the period range considered) design eccentricity expression, which is independent of the uncoupled lateral period, have been

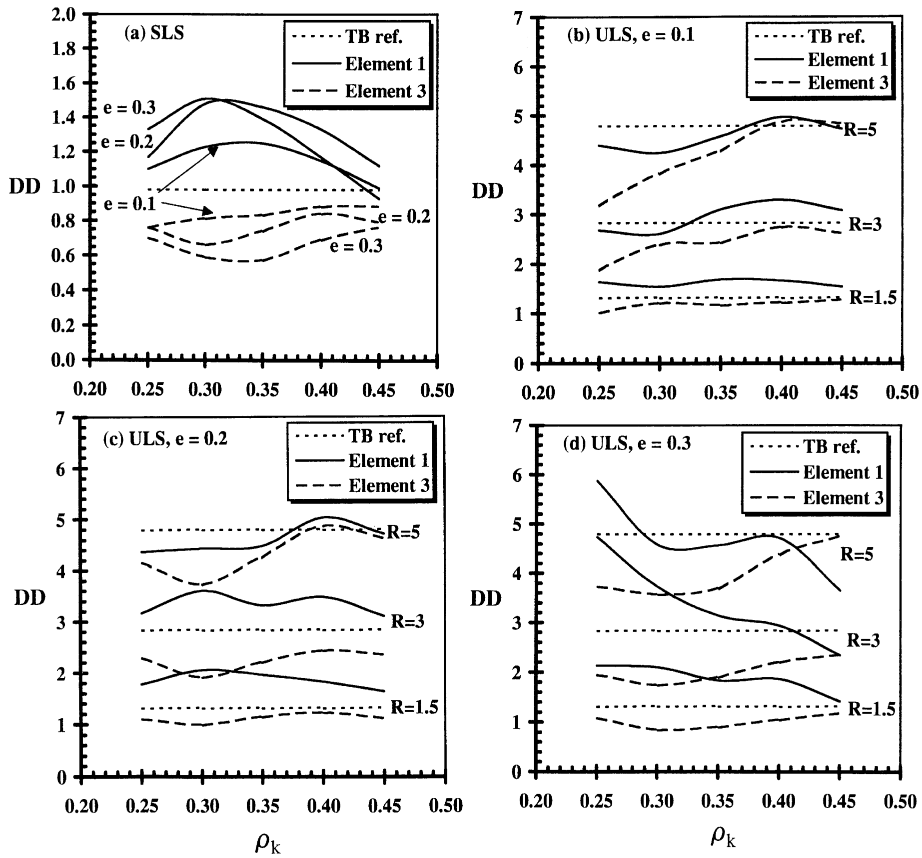

Figure 12. Performance of the mass-eccentric TU structural model, in both limit states

provided for design purposes. The required overstrength factor has also been investigated in order to result in the responses of both edge elements being lower than, or in the neighbourhood of, those of the TB reference model. The proposed design procedure may be considered to have the following four advantages.

1. It is a closely optimized design procedure. When the proposed optimized design eccentricity expression and the proposed overstrength factor are employed in determining the strengths of resisting elements, the responses of both edge elements are nearly equal in value, nearly constant across the full range of ρ_k, and are lower than, or in the neighbourhood of the response of the TB reference model. The overstrength factor proposed in this paper is substantially lower than that corresponding to UBC-94 and NBCC-95. Compared with the overstrength factor resulting from EC8-93, the recommended values are marginally higher when ρ_k ranges from small to moderate, and are marginally lower when ρ_k ranges from moderate to large. Hence, the proposed procedure not only leads to nearly optimal, satisfactory and consistent performance of TU structures, but it also requires a much smaller overstrength factor, compared with those resulting from UBC-94 and NBCC-95.
2. It has taken into account the influences of four primary system parameters, e, ρ_k, R and T_y. The first three primary system parameters have been considered explicitly in the design eccentricity expression. The influence of T_y is considered implicitly, in terms of an 'averaged' (over the period range covering

short, intermediate and long-period buildings) design eccentricity expression. Hence, the recommended design procedure can consistently result in nearly optimal performance of TU structures over a wide range of combinations of the primary system parameters.

3. It retains simplicity and can be easily implemented in design practice. It can be easily implemented for design of single-storey asymmetric buildings and a special class of multistorey asymmetric buildings having proportional stiffness properties amongst their lateral load resisting elements parallel to the direction of ground motion input and having their mass centres of floors located along a vertical axis. For these two categories of asymmetric buildings, the normalized static eccentricity, e, and the normalized stiffness radius of gyration, ρ_k, can be simply calculated by well-established procedures. The recommended optimized design eccentricity and the overstrength factor can be easily read from design charts. Furthermore, for each limit state, only one elastic, static structural analysis is required, due to the proposal of a single, optimized design eccentricity expression.
4. Its applicability has been generalized to TU systems with and without transverse elements. Since these two structural modelling considerations are the two extremes of simplification, the true response of actual buildings under bidirectional seismic loading will be between the bounds predicted by the above two structural models, as recently stated by De La Llera and Chopra.[15] This paper has demonstrated that for both structural models, the recommended design procedure leads to closely optimal, satisfactory and consistent performance for both limit states.

It should be noted that the recommended design procedure cannot yet be extended accurately to TU structures in which the distribution of mass is significantly unbalanced. This issue remains to be addressed in future research studies.

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